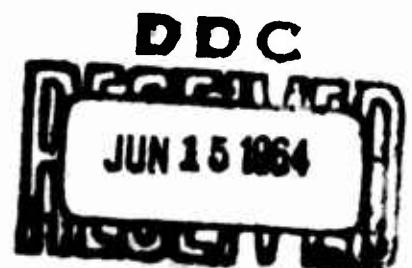*20-p $0.50*

GAMES AND SIMULATIONS

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PREFACE

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SUMMARY

The prevailing pessimism concerning the usefulness of game theory for military analysis is premature, and the time has arrived for a second look. The advantages in a game theoretic analysis of a military problem are large. These include the possibility of dealing with a wide spectrum of strategic possibilities, the explicit inclusion of the basic two-(or many-) sidedness of military conflicts, and the opportunity for optimization. However, for some time to come, game theoretic analysis cannot be expected to deal with the staggering complexity of large-scale military engagements. Progress will be made primarily with highly simplified models. As a result game theoretic studies will need supplemental "unpacking" and "testing." Imbedding abstract games in a family of models whose aggregated solutions are refined and evaluated by more detailed simulation appears to be one promising direction.

GAMES AND SIMULATIONS

1. INTRODUCTION

A basic difficulty with large scale simulation in any military field is the relatively narrow range of cases that can be examined. This results from the large number of parameters needed to define interactions and the many variables defining doctrine. In addition, the process involved is two-sided; that is, the simulation deals with a conflict situation where one side has only partial control of the course of events. As a result, the analyst can select only a very small sample out of a vast space of possibilities. It is difficult in many cases to determine whether this sample is even near the region of greatest interest. Certain kinds of extremal arguments ("worst cases," dominance, "insurance strategies,") can be used to reduce the complexity, but these are usually feeble measures in light of the enormous range of cases that could be explored.

On the other hand, there are telling reasons why detailed simulation is resorted to in many military analyses. Probably the most compelling is the fact that in many instances phenomena are understood rather well in the small--the operation of an aircraft or a missile, the damage effects of a bomb, the kinematics of an interceptor, the discrimination capabilities of a radar, etc.--but the over-all effect of the interaction of a complex set of such phenomena is not easy to grasp. This is particularly the situation where new, possibly non-existent, weapons are concerned, and where no operational experience exists. Engineering analyses may circumscribe some of the system characteristics, but overall effectiveness depends on a very complex process. The process can be followed in great detail on a computer, indicating interactions that

would otherwise be too intricate to think through. In this situation a large-scale simulation can be employed to test hypotheses arrived at by some other route. It can also be used for exploratory experimentation; by running and analyzing a number of specific cases, some insight into the overall process can be obtained.

It is general practice to supplement simulations with other kinds of analysis--qualitative argument, "paper and pencil," analytic models, and so on. The supplemental analyses are more aggregated and approximate, but also more rapidly computable so that a wide range of cases can be examined to some extent. Unless a simulation can be embedded in a context of informal analysis of this sort, it will ordinarily suffer a loss of credibility. This fact suggests that what is needed for the full exploitation of computer simulations is a family of models, a set of analyses of varying levels of generality, where models at the higher levels of aggregation can be used to suggest "hypotheses"--preferred doctrines, preferred weapon mixes, etc.--which can be "tested" by analyses of more detail but less power.

Viewed in this light, as a component of a multi-leveled model structure, a tool that has been somewhat neglected in recent years begins to show renewed promise. This tool is game-theoretic analysis.

2. SOME HISTORY

In the early days at RAND, a large effort went into the theory of games. Major contribution to the theory itself were made, as well as widespread attempts at application to military problems. The theory looked extremely promising for a number of reasons. It was the first model which appeared conceptually suitable for con-flict situations--processes where the variables were partially under control of an opponent. The notion of solution in the zero-sum case also was very satisfying

from a military point of view. It contained the notion of optimization, but also of a _safe_ strategy--i.e., if you played a minimax strategy and the enemy played in a non-optimal fashion, you could only gain by his mistake. The minimax solution had a very solid form of stability. It was completely non-cooperative, and, if you presumed the enemy was as perceptive as yourself, you could announce your strategy beforehand without loss of expected gains.

Unfortunately, the initial enthusiasm died down rather quickly. There were at least two reasons. One was purely technical; games with interesting military content turned out to be exceedingly intractible. No very powerful algorithms for computing optimal strategies in complex games turned up. This was true for even quite simple games where the number of variables in the strategy space was small. For games with any richness at all--e.g. games with both attack and defense on both sides--the problem looked utterly hopeless even with the aid of the most powerful computers.

The other reason was more conceptual. Military games, except for tactical or battle situations, are generally not zero-sum. Especially in general nuclear wars, both sides can lose disastrously. The theory of solution for non-zero-sum games was not nearly so satisfactory from a military point of view. Usually some form of cooperation was required--which did not seem to be at all appropriate to such a non-cooperative activity as war.

These two reasons acted as strong depressants, and activity on the military applications of the theory of games died down to a trickle.

A few hardy souls maintained some activity, however, with non negligible results. At any early date general attack-defense games were solved (1). A theatre air-war game was played first on an analogue computer, then transferred to an IBM 701 digital computer where a large sample of strategies on each side was played through and

preferred strategies found by inspection (2). Finally, a
simplified form of the game was solved analytically (3, Chap. 10.)
Several duels involving the timing of missile fire have been
solved (4,5). Recently promising attacks have been made on the
missile-antimissile duel. These results and others like them,
although not adding up to an overwhelming story for game
theory, indicate that in the areas of tactical interactions
where zero-sum analysis appears appropriate, the situation
is by no means hopeless.

For some time to come, however, game theoretical
techniques are not likely to have the power required to
deal with complicated models. We can expect that their
use will be restricted to aggregated games couched at a
fairly high level of generality. Of course, there would
be absolutely nothing wrong with this situation providing
the theory of military interactions were sufficiently well
developed so that the simplifications could be justified
on scientific grounds. Unfortunately that is not the case
at present. Military science is still in an undeveloped
state. Conclusions reached by game theoretic analysis
can only be considered as hypotheses requiring farther
testing. We can't hope for demonstrative tests. The
ultimate test--actual military engagements--is much too
rare; field tests and maneuvers are suspect as simulacrums
of war, and usually ambiguous in their results. Additional
support for the conclusions from game-theoretic analysis can
be obtained by detailed simulation. It would be a mistake
to take the simulation for reality, but at least the simula-
tion can indicate whether the conclusions from the general
study are negated by omitted or aggregated details.

3. MODEL FAMILIES

We are experimenting at RAND with a model family of
the sort I have been suggesting in the area of strategic
war planning. We are putting together a set of three
models, with the basic aim of generating a detailed war plan.

At one end of the scale (see Fig. 1) is a routine (called STRAP for Strategic Actions Planner) which expands a short summary of a desired plan into a highly detailed matching of weapons and targets taking into account specific operational constraints such as generation and flight times, bomber-tanker match, attrition patterns over enemy territory and the like. At the other end of the scale is a highly aggregated, two sided war game (called STROP for Strategic Optimizing Planner). For some initial conditions an explicit solution to the war game will be possible. For others, it may be necessary to resort to a sampling procedure. The game is designed so that it will run a single case (a single strategy pair) in a fraction of a second on a higher speed computer so that a very large sample can be run in a few minutes. In initial test runs, for example, a 160 x 160 matrix, about 25,000 cases, required 2 minutes to run and reduce. The reduction consisted of eliminating all dominated strategies on each side. In the test cases, reduction by dominance was enormous--the routine often wound up with unique undominated strategies. In no case was there more than six undominated strategies on either side.

At the intermediate level is a routine we call STRIP (Intermediate Planner) STRIP is a two-sided simulation whose primary purpose is to unpack a preferred pair of strategies from STROP in terms of time and geography, and make a reassessment of the outcome of the interaction of those two as guidance for the preparation of a summary to be fed to STRAP.

The three models can be employed separately, or when completely implemented, as a single package, the results of one model feeding directly into the next lower level.

4. PAYOFF FUNCTION FOR GENERAL WAR

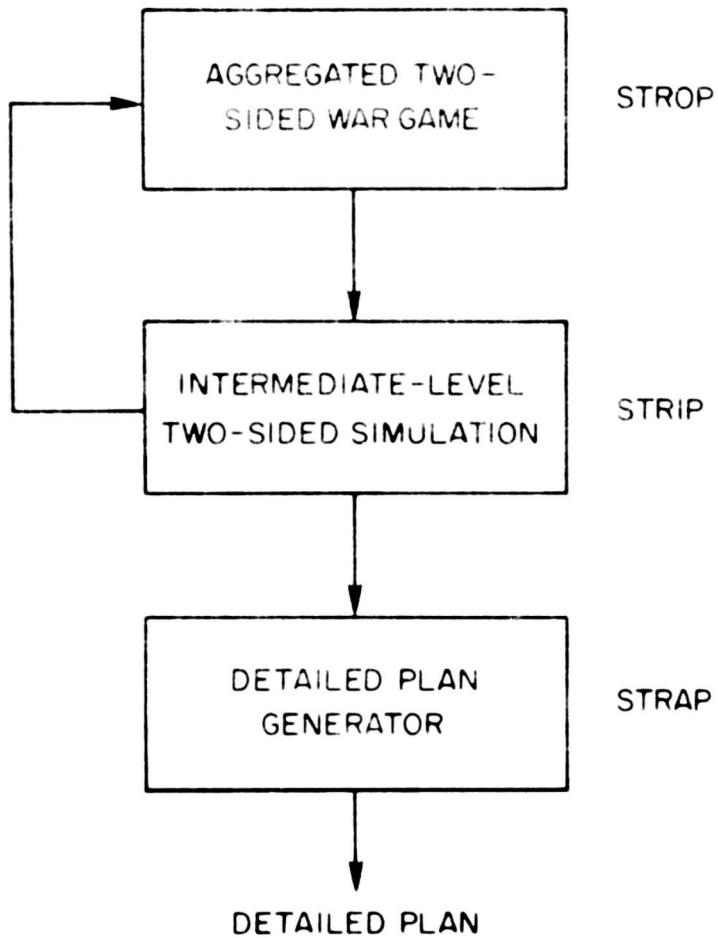A crucial problem in designing a general war game such as STROP is the specification of pay-off function.

FIG. 1 — PLANNING MODEL FAMILY

As I noted earlier, the non-zero-sum character of general war creates conceptual difficulties with the notion of solution. In addition, general war with thermo-nuclear weapons may eventuate in catastrophic destruction of both sides. This has a tendency to make any strategic considerations seem irrelevant.

Fig. 2 is a commonly used diagram to present the payoff situation in a general nuclear war. Each side prefers more damage to the other side and less to itself, as indicated by the arrows. However, for each side there is a level of damage which can be considered disastrous. The critical level has been variously defined as the level at which it is impossible to recuperate within a reasonable time, or the level at which a nation is reduced to a second-rate power, or the level at which war-making potential is negligible. In whatever way the critical levels are defined, the presumption is that the side suffering that damage has essentially lost the war irrespective of what happens to the other side.

Given the forces available to the U.S. and the Soviet Union at present and for the next several years, war games have a tendency to wind up with both sides in the catastrophic region. This is not necessarily undesirable from the standpoint of deterrence, but it gives no strategic guidance in case deterrance should fail. Actually the situation is a little worse. Table 1 presents a highly simplified type of anaysis which dramatizes the dilemma posed by each side having the capability of destroying the other.

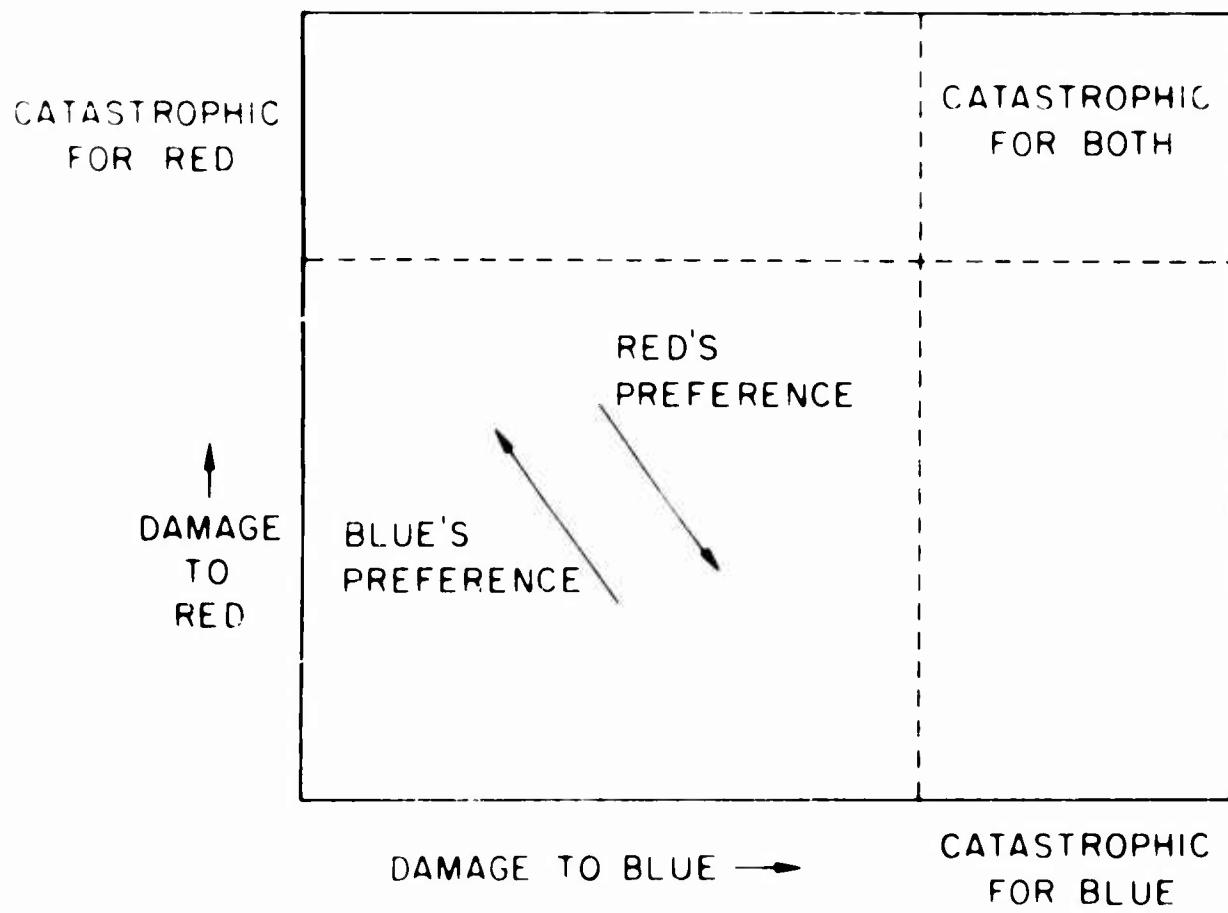| RED BLUE	CF	CV
CF	0 0	- • W
CV	W - •	- • - •

TABLE I

FIG. 2 -- PAYOFF SPACE

The assumption underlying the table is that if one side utilizes most of its forces for attacking the other side's weapons (CF for counterforce), this is not sufficient to preclude catastrophic damage if the enemy elects to concentrate on counter-value (CV) targets. The table is a simplified payoff matrix where the options open to either side is to concentrate either on counterforce or countervalue attacks. The upper notations in these boxes refer to the payoff to Blue, the lower figures the payoff to Red. When the situation is roughly symmetrical, if both sides elect CF they come out about even. If Blue elects CF and Red elects CV, Blue cannot prevent catastrophic damage to himself ($-\infty$) whereas Red is in a fairly good position (W for "win"). The converse is true for elections of CV for Blue and CF for Red. Finally, if both elect CV they wind up with mutual destruction. As far as Blue is concerned if Red elects CF, Blue prefers CV, and if Red elects CV, Blue loses nothing by playing CV. In short, the countervalue option dominates the counterforce option. The same is true for Red. Presumably, in the absence of a very strong agreement, if war occurs both sides will prefer a countervalue attack and mutual destruction will result.

This simple type of analysis has been employed, to indicate that central nuclear war is "irrational," and therefore unlikely. It has also been used to "demonstrate" that if war does occur, both sides will be destroyed.

What is missing from the approach is a serious consideration of the critical level of damage. If such levels exist, there should be a significant difference in evaluation of the outcome depending on which side of the critical level the damage falls. In the lower levels of destruction the assumption that one side will be willing to trade damage to enemy targets for los s of its own at some exchange ratio appears reasonable. But, as the level of damage approaches the critical value, the exchange ratio should increase, and

presumably no amount of damage to the enemy can compensate for exceeding the critical level. An equal-value curve in the payoff space should be concave upward and become asymptotic to the critical value line as in Fig. 3.

I call the statement that the relative worth of value targets lost to enemy targets destroyed accelerates as a critical level of damage is approached the <u>assumption of increasing concern</u>. There are, of course, an infinite number of mathematical expressions of the payoff which have the properties mentioned. An especially simple expression is

$$P_B = D_R - D_B - \frac{A}{\lceil C_B - D_B \rceil} \tag{1}$$

where P_B is the payoff to Blue, D_R is the damage to Red value targets, D_B the damage to Blue. C_B is Blue's critical level and A is a scaling factor indicating how rapidly the relative value accelerates. The corner quotes indicate that $\lceil C_B - D_B \rceil$ is to be taken as 0 if $C_B - D_B$ is negative. The payoff to Red would be the same expression with the subscripts interchanged,

$$P_R = D_B - D_R - \frac{B}{\lceil C_R - D_R \rceil}$$

where B is Red's "concern factor."

Another way to look at the concern factor is that in reality the critical level will not be well-known, either for ourselves or for the enemy. On the other hand as the damage level increases there is an increasing probability that the critical level has been exceeded. The concern factor, then, expresses the effect of this increasing probability, where the A's and B's can be taken as rough measures of the degree of uncertainty we have about the critical values.
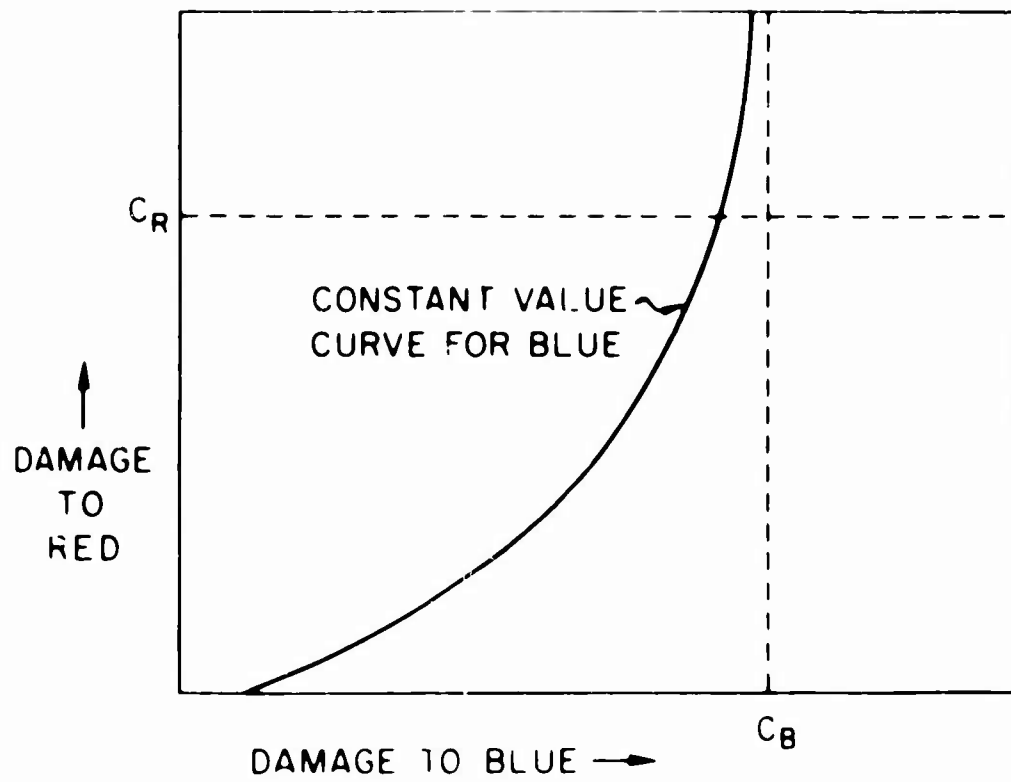
FIG. 3 — ASSUMPTION OF INCREASING CONCERN

5. APPLICATION TO A SIMPLE WAR GAME

The effect of the assumption of increasing concern on
he nuclear dilemma posed earlier can be indicated by a
simple general war game involving one kind of weapon and
one kind of value target. The game can be interpreted as a
counterforce exchange, with both sides determining their
payoff from the damage potential of remaining forces, or
as an all-out attack on both counterforce and countervalue
targets, with the payoff determined by actual destruction.
The structure of the game is unchanged by these inter-
pretations, but the relative effectiveness of weapons in
the counterforce mission would differ; the effectiveness
should be somewhat higher in the withholding case. In
this simple game the only strategic variable on each side
is the proportion of weapons allocated to the counterforce
mission.*

Fig. 4 shows the payoff matrix for a symmetric game
where the counterforce effectiveness for each side is
assumed to be quite low--the probability of a weapon
destroying an enemy weapon is .2. The other parameters are
set more or less arbitrarily for illustration. The total
forces on each side are 150 (in units of damage potential
to value targets) the critical values (C_B, C_R) are 100 and
the concern factor (A,B) 1000. In this case, if either side
allocates all its forces to counterforce, the other side can
still impose critical damage (indicated by $-\infty$ in the matrix)
by allocating most of his forces to countervalue. We thus
have all the ingredients of the nuclear dilemma. Never-
theless, inspection of the matrix shows that there is a
point (indicated by a star in the figure) x = y = .375
with the property that if Red allocates .375 of his forces
to counterforce, then Blue has a maximum of his payoff at

*For a more complete exposition of the model see (6).

the corresponding allocation of Blue forces, and conversely,
if Blue allocates .375 to counterforce, then Red has a
maximum at the same allocation.

Thus, there is an equilibrium point in pure strategies
for the game, where neither side suffers critical damage.
The equilibrium point is non-cooperative in the sense that
if one player elects to play at that point, than the other
maximizes his payoff by also playing at that point and vice
versa.

The existence of the equilbrium point does not depend
upon symmetry. The symmetrical case was selected as an
example because it involves the nuclear dilemma in its
sharpest form. For both the symmetric and assymetric cases,
a similar equilibrium point exists, providing both sides
have a non-negligible counterforce capability--i.e. providing
the kill probability of a weapon on an enemy weapon is not
zero.*

6. CONCLUDING REMARKS

The illustrative game I have presented is a very simple
one, designed primarily to show the increased possibilities
resulting from adding a small amount of structure to the
payoff function. STROP has somewhat more content including
both missile and bomber forces, bomber defenses, and relative
timing of attacks. This additional structure, however,
does not change the overall equilibrium analysis.

The particular payoff function employed is probably
a vast overimplification of national aims in nuclear
conflict. Before taking the analysis too seriously, a
great deal of study is needed to assure the reality of
critical levels, and the existence of increasing concern
with expected damage. At the moment the most that can be

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*In the symmetric case a relatively simple expression
can be obtained for the equilibrium point. An analytic
solution also exists for the non-symmetric case, but is somewhat
complicated. A computer routine has been programmed to evaluate
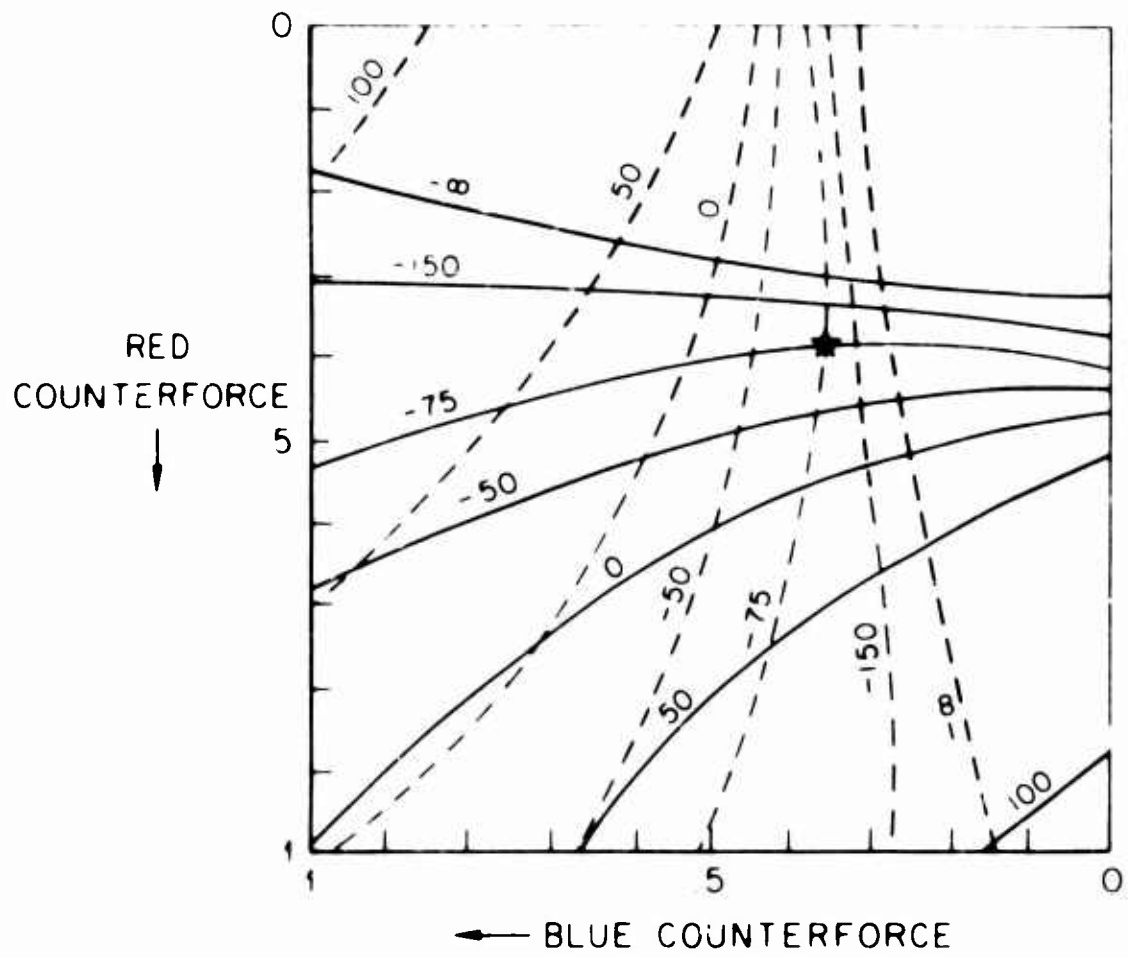the equilibrium point allocation for the non-symmetric case.

FIG 4 — PAYOFF MATRIX, SYMMETRIC GAME

said is that the payoff function embodies many of the factors which arise in discussions of nuclear conflict, and that formal expression of those elements in the assumption of increasing concern transforms the nuclear dilemma in a crucial way--leading to a stable solution rather than an unresolved paradox of mutual disaster.

In analyses concerned with the value of defensive systems, it is clear that some way of devising a trade-off (not necessarily constant) between attack and defense is necessary. The payoff function I have suggested does give a preliminary handle on this problem. No essential change is made if, for example, trade-off factors are introduced indicating the relative worth to Blue of the loss of a Blue value target vs. the distruction of a Red target, and similarly for Red. In this way an approach is opened for examining the stabilizing (or distabilizing) effects of changes in defense level.

Finally, I would like to reiterate a point made earlier, namely that the abstract game is not designed to stand by itself, but needs the back-up of more detailed models. The detailed model STRAP has been coded and checked out; the intermediate model STRIP is being coded. Within the next months it will be possible to see how well the recommendations from STROP hold up under specific operational constraints.

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